Lower-Upper Implicit Scheme for High-Speed Inlet Analysis

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Abstract

A NUMERICAL method is developed to analyze the inviscid flowfield of a high-speed inlet by the solution of the Euler equations. The lower-upper (LU) implicit scheme in conjunction with adaptive dissipation proves to be an efficient and robust nonoscillatory shock-capturing technique for high-Mach number flows as well as for transonic flows.

Contents

Recent interest in the aerospaceplane and other hypersonic vehicles revitalized the research on high-speed propulsion systems. In the design of supersonic and hypersonic propulsion systems, the analysis of high-speed flow through an inlet plays a critical role. The turboramjet inlet flowfield includes the incoming supersonic flow deflected by oblique shock waves and the subsonic diffuser flow after the terminal normal shock wave, while the scramjet inlet flow is characterized by strong oblique shock waves. The Euler equations, which represent the hyperbolic conservation law, can be a useful testbed for developing and evaluating a shock-capturing numerical algorithm. In parallel with the developments in upwind schemes, it has been found that steady aerodynamic flows containing moderately strong shock waves can be satisfactorily predicted by a central-difference scheme augmented by a carefully controlled blend of first- and third-order dissipative terms.[1] In this paper, the performance of adaptive dissipation is demonstrated for strong oblique shock waves in high-Mach number flows on a near-uniform mesh.

Let x, y, and t be Cartesian coordinates and time. Then for a two-dimensional flow, the Euler equations can be written as

$$\frac{\partial W}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \qquad (1)$$

where W is the vector of dependent variables and F and G are convective flux vectors.

In order to suppress the tendency for spurious odd and even point oscillations and to prevent nonphysical overshoots near shock waves, the finite-volume scheme is augmented by artificial dissipative terms. The dissipative term, which is constructed so that it is of third order in smooth regions of the flow, is explicitly added to the residual. For the density equation, for example, the dissipation has the form

$$d_{i+\frac{1}{2},j}-d_{i-\frac{1}{2},j}+d_{i,j+\frac{1}{2}}-d_{i,j-\frac{1}{2}}$$

where

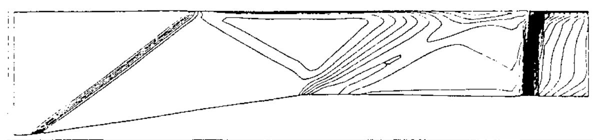
$$d_{i+\frac{1}{2},j}=\epsilon^{(2)}_{i+\frac{1}{2},j}(\rho_{i+1,j}-\rho_{i,j})-\epsilon^{(4)}_{i+\frac{1}{2},j}(\rho_{i+2,j}-3\rho_{i+1,j}+3\rho_{i,j}-\rho_{i-1,j}) \qquad (2)$$

Let S be the cell area, which is equivalent to the inverse of the determinant of the transformation Jacobian. Both coefficients include a normalizing factor $S_{i+\frac{1}{2},j}/\Delta t$ proportional to the length of the cell side, and $\epsilon^{(2)}_{i+\frac{1}{2},j}$ is also made proportional to the normalized second difference of the pressure

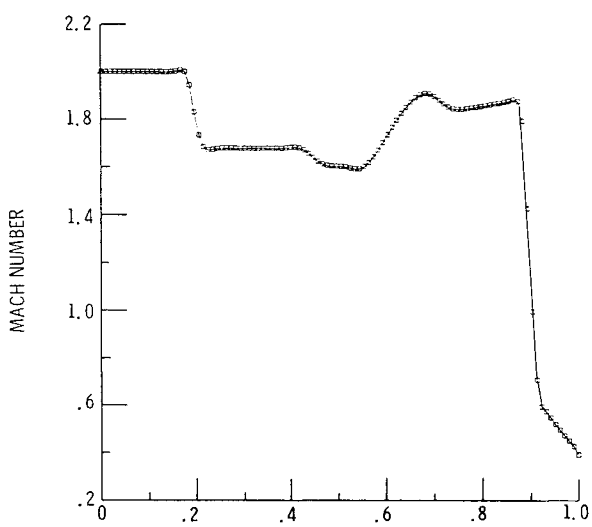
$$\nu_{i,j}=\left|\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{p_{i+1,j}+2p_{i,j}+p_{i-1,j}}\right| \qquad (3)$$

in the adjacent cells. The first-order terms are needed to control oscillations in the neighborhood of shock waves and are activated when strong pressure gradients in the flow are sensed.

An unconditionally stable implicit scheme that has error terms of the order $(\Delta t)^2$ at most in any number of space dimensions can be derived by the LU factorization.[2] Let D_x^- and D_y^+ be backward-difference operators and let D_x^+ and D_y^+ be forward-difference operators. Then the LU implicit scheme



a) Mach number contours.



b) Mach number along the centerline.

Fig. 1 Mach 2 inlet with the terminal shock wave.

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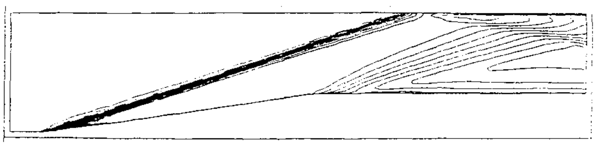
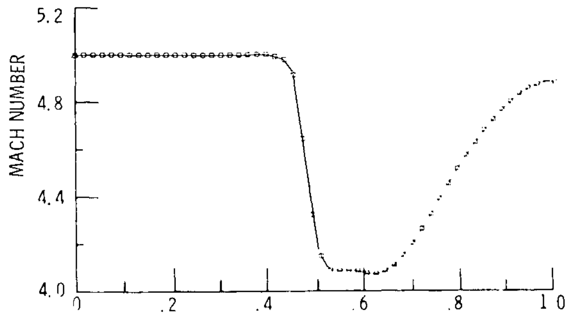


a) Mach number contours.



b) Mach number along the centerline.

Fig. 2 Mach 5 inlet.

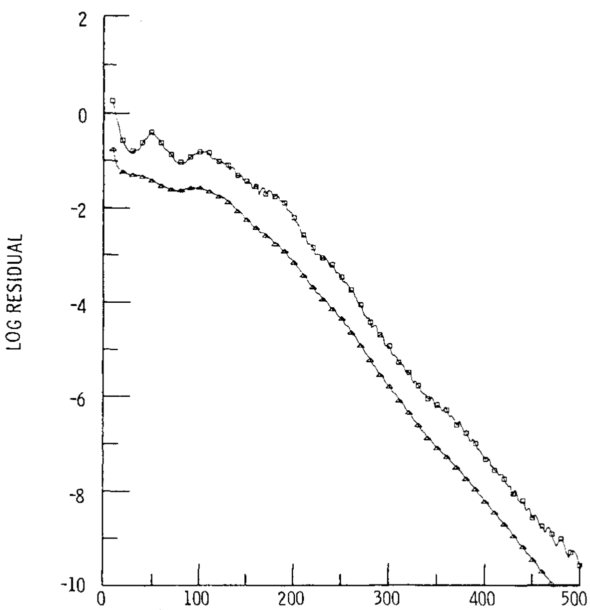


Fig. 3 Convergence history (Mach 5 inlet).

becomes

$$\{I+\beta\Delta t(D_x^- A^+ + D_y^- B^+)\}\{I+\beta\Delta t(D_x^+ A^- + D_y^+ B^-)\}$$

$$\delta W+\Delta t(D_x F+D_y G)=0 \tag{4}$$

where D_x and D_y are central-difference operators.

Here, A^+, A^-, B^+, and B^- are constructed so that the eigenvalues of $(+)$ matrices are nonnegative and those of $(-)$ matrices are nonpositive:

$$A^+ = \tfrac{1}{2}(A+r_A I), \qquad A^- = \tfrac{1}{2}(A-r_A I)$$
$$B^+ = \tfrac{1}{2}(B+r_B I), \qquad B^- = \tfrac{1}{2}(A-r_B I) \tag{5}$$

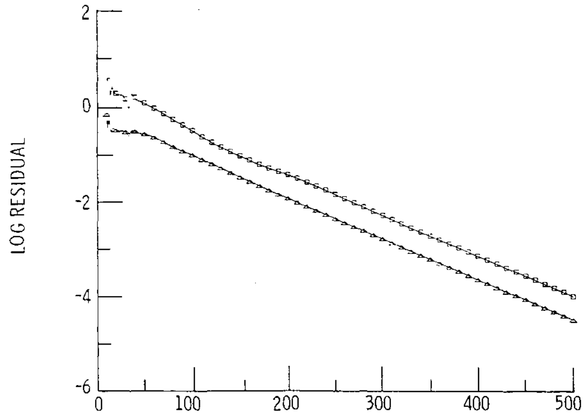


Fig. 4 Convergence history (Mach 20 inlet).

where

$$r_A \ge \max(|\lambda_A|), \qquad r_B \ge \max(|\lambda_B|) \tag{6}$$

Here, λ_A and λ_B represent the eigenvalues of Jacobian matrices. Equation (4) can be inverted in two steps.

Two-dimensional calculations have been performed on a 54×32 H-mesh for a schematic high-speed inlet. The inlet ramp angle is 9 deg, and the shoulder angle is set to 0.5 deg for the terminal shock wave problem. At the inflow boundary, all the flow quantities are specified, and they are extrapolated from the interior at the outflow boundary for supersonic outflow. For the terminal shock wave problem, the pressure is prescribed at the outflow boundary. The Mach number contours and Mach number along the centerline are shown in Figs. 1 and 2. The terminal shock wave problem with a freestream Mach number of 2 is shown in Fig. 1. Figure 2 is for a supersonic throughflow with a freestram Mach number of 5. At the figures shown, wave structures observed in high-Mach number flows such as oblique shock waves, reflected shock waves, expansion fans, and the interaction of shock waves with expansion fans are successfully predicted. The convergence histories in Fig. 3 (Mach 5) and Fig. 4 (Mach 20) show that the residuals drop linearly and continuously, which demonstrates the efficiency of the present numerical method. The two indicators in the convergence histories are the logarithm of the maximum and the average density residuals. However, as the Mach number is increased, the convergence rate is hindered. Recent study shows that it is possible to improve the accuracy of shock waves by using a total variation diminishing scheme while exhibiting comparable computational efficiency.[3]

References

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